The University of Michigan

Department of Mathematics

East Hall, 525 East University • Ann Arbor, MI 48109-1109
Phone: 734-764-0335 • Fax: 734-763-0937
http://www.math.lsa.umich.edu/
E-mail: ralf@math.lsa.umich.edu

December 19, 2001

Postdoc CNLS Programs
Center for Nonlinear Studies
P.O. Box 1663
MS B258
Los Alamos, NM 87545

Dear Members of the CNLS,

I would like to apply for a postdoctoral research position in the Center for Nonlinear Studies at Los Alamos National Laboratory, or for any other appropriate visiting positions.

I am currently an Assistant Professor in the Mathematics Department at the University of Michigan, Ann Arbor; I completed my Ph.D. in the Program in Applied and Computational Mathematics at Princeton University in 1998 under the supervision of Professor Philip Holmes.

As I have indicated in an accompanying research statement, I have diverse interests in applied mathematics, particularly in dynamical systems and differential equations. I am especially interested in the analysis, numerical simulation and modelling of complex spatial and temporal dynamics and pattern formation in nonlinear spatially extended systems, particularly those occurring in physical applications such as fluid dynamics, and in biological systems.

In support of my application, I have enclosed a curriculum vitae, which includes a list of publications, and a statement of my past research and continuing research interests. I have requested that letters of recommendation be sent to you by my advisor, Professor Philip Holmes (Princeton) and also by Professors David McLaughlin (Courant Institute, New York University), Charles Doering (Michigan) and Robert Krasny (Michigan). Thank you for your consideration of this application.

Yours sincerely

RWittenberg

Ralf W. Wittenberg

The University of Michigan

Department of Mathematics

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Letter of Recommendation for Ralf Wittenberg

December 12, 2001

I am writing in enthusiastic support of Ralf Wittenberg's application for a tenure-track position in your department. Ralf asked me to write a letter which specifically addresses his teaching history here at the University of Michigan. I am currently the Associate Chair for Education in the Department of Mathematics. Among the dozens of 3-year assistant professors who have been at Michigan in my time here, Ralf has made one of the most impressive contributions to our educational program. He has taught courses at all levels of our curriculum and has been a resounding success at all these levels. A brief review of his record here at Michigan will make it clear why I am certain he will be a very valuable addition to the educational mission of your department.

I will go through each of his semesters here and discuss his contribution. In the fall of 1999, his first semester at Michigan, he taught 2 sections of Math 156 (Applied Honors Calculus II). This class is a honors class developed recently by Robert Krasny for Engineering and Applied Mathematics students who enter the university with advanced AP credit. It is a challenging and rigorous course. He received instructor ratings of 3.9 and 4.1 (out of a possible 5.0), which are quite good for a Freshman course. I looked over all the written comments for the course and many students commented on how helpful Ralf was. One of them signed themselves simply "An inspired Mathematics student."

In the Winter of 2000, Ralf taught 2 sections of Math 216(Differential Equations). This class is taught in large lectures of around 100 students together with a recitation/computer lab run by a graduate student. He received outstanding instructor ratings of 4.2 and 4.7 in a course which is often unpopular with students. In the written comments, several students referred to him as the best mathematics teacher they had ever had. Typical comments include "He presents material/concepts in an organized and understandable manner and truly cares if the class learns" and "I actually found myself doing eigenvalues for fun in other classes." Many other students referred to his enthusiasm for the material and his approachability. After the course was over he volunteered to help to rewrite some of the computer labs (which the students do using MATLAB) and these changes have been incorporated into subsequent versions of the course.

In Fall 2000, he taught the only 2 sections of Math 256(Applied Honors Differential Equations). He received phenomenal instructor ratings of 4.9

and 5.0 (out of a possible 5). He developed all of his own material for this course, including computer labs and a brief introduction to MAPLE (which has also been used by other instructors.) Comments included "This class was AWESOME!" and "Ralf is the man. He is one of the best teachers I've had."

In Winter 2001, he taught a graduate course Math 656 on Partial Differential Equations. This course had a more theoretical focus. His instructor rating was an impressive 4.7 in this course. (As is common in a small graduate class there were no written comments on the evaluations.)

This semester Ralf is teaching Math 454(Boundary Value Problems for Partial Differential Equations) and Math 156(Applied Honors Calculus II). Math 454 is largely taken by students from the Engineering College and the course is focussed on applications. Next semester he will be teaching Math 454 and Math 471(Introduction to Numerical Methods). Math 471 involves extensive use of technology by both instructor and students. It is too early to have received any evaluations yet on these courses, but I expect that they will be as impressive as ever.

As you can see, Ralf has been a very successful educator at all levels of our program. He has taught both applications-oriented and more theoretical courses with equal aplomb. He is very comfortable and adept at the integration of technology into the curriculum. I recommend him to you without reservation.

Sincerely,

Richard D. Canary
Associate Chair for Education
Professor
Department of Mathematics
University of Michigan

The University of Michigan

Department of Mathematics

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E-MAIL: doering@umich.edu

23 December 2001

Re: Ralf Wittenberg

This is a letter of reference for Dr. Ralf Wittenberg, who has applied to you for a position. Ralf has been a postdoctoral assistant professor (non-tenure track) here in the University of Michigan's Department of Mathematics since the fall of 1999. Ralf has participated in my research working group meetings during that time and I have become familiar with several aspects of his research program. Most recently he has collaborated closely with me on a particular project. More on that below.

Ralf's central research focus going back to his Ph.D. dissertation work has been the numerical and analytical investigation of spatio-temporal chaos. The Kuramoto-Sivashinsky (KS) equation is one well-characterized paradigm of such phenomena, and Ralf has applied a number of sophisticated methods of modern applied and numerical analysis to it. I was particularly impressed by his study of the long-standing and extremely frustrating open question of *a priori* estimates for the solutions of the Kuramoto-Sivashinsky equation. The fact is that every numerical simulation has suggested that the L^∞ norm of solutions remains $O(1)$ as the length L of the solution interval is increased (or equivalently, the L^2 norm of solutions scales at most like $L^{1/2}$). The problem is that nobody has ever been able to prove directly from the pde that this scaling is an upper limit; the best estimates are much higher powers of L. Ralf showed that the addition of a linear, lower order perturbation to the KS equation produces solutions which violate the observed unperturbed scaling and approach much closer to the best rigorous estimates. So although Ralf did not solve the problem, in my mind he has shown that it is much more subtle than it appears. In some sense, in regard to the asymptotic scaling of norms on long intervals, the KS equation is not structurally stable with respect to such low order perturbations. I invited Ralf to submit this paper to the journal I edit, *Physics Letters A*, and after thorough review I accepted it for publication (subject to minor revisions).

Ralf's recent project with me has been the study of turbulent Rayleigh-Benard convection in the Boussinesq equations with the goal of extracting rigorous limits to the heat transport (the Nusselt number Nu) as a function of the applied temperature difference (the Rayleigh number Ra). This is a well developed and active field of research, both theoretically and experimentally. Our twist on the existing approaches has been to include more realistic—in the context of experiments—boundary conditions. We have generalized the model to include less-than-perfectly thermally conducting boundaries and showed how one can deduce rigorous estimates. Although this work has not yet led to a resolution of the major open problem in this area of mathematical fluid dynamics, i.e., that the best bounds on Nu scale with a higher power of Ra than is observed in experiments, it yields some interesting insights into the role that the boundary conditions can play in the analysis.

All in all, Ralf is a creative and well-trained applied mathematician with a tremendous range of computational and analytical tools at his disposal. I expect that his research will have lasting impact. Ralf has carried a full teaching load (two 3- or 4-credit hour courses at a time) all but one semester while he has been at Michigan. Ralf takes these duties very seriously, and he has worked hard to develop his teaching skills at the complete range of levels, from lower division to graduate. This big responsibility consumed much of his attention here, and consequently his written output has not been as high as I would have hoped during this period.

Sincerely,

Charles R. Doering
Professor of Mathematics

1-9-02

PACM
The Program in Applied
and Computational Mathematics

Princeton University
Fine Hall, Washington Road
Princeton, New Jersey
08544-1000 USA

December 19, 2001

To whom it may concern:

I am writing in support of RALF WITTENBERG's application for a position in your Department. Ralf is completing an Instructorship at the University of Michigan which he took following a Postdoc at IMA Minnesota. Prior to that he worked with me from 1994 until his graduation from Princeton in 1998. In this period he did two major pieces of work: a careful study of the validity of center manifold and normal form reductions of a reaction-diffusion (RD) equation near a codimension two bifurcation point (Physica D, 1997); and numerical and analytical studies, using wavelet decompositions, of spatiotemporal chaos in the Kuramoto-Sivashinsky (KS) equation, leading to the development of `local models' (Chaos, 1999 and Nonlinear Dynamics, 2001 [the latter paper was held up for over a year as part of a special issue]). With Jonathan Mattingly, a fellow PACM student, he also wrote notes on a lecture course I gave at the Newton Institute and did the bulk of the work in preparing two extensive review articles (Physics Reports, 1997 and NATO ASI Proc., 2001).

The RD normal form work was very nice, producing an authoritative paper, over 75% due to Ralf, but the second problem, which formed Ralf's thesis, was and is much deeper and more difficult. The procedures for reducing spatially-confined and near-critical evolution equations to (small) ODE and even PDE (amplitude) systems is well-established, but much less is known about spatially extended systems. Can one really treat them as weakly coupled `arrays' of constrained systems? How should one `replace' the large scales omitted in reduction to a spatially-localised model? What are the key interactions and energy transport mechanisms among spatial scales?

Mathematically precise versions of, and answers to, these questions would significantly advance our understanding of numerous pattern forming systems (granular flows, chemical reactions, morphogenesis, turbulence, ...), not to mention numerical schemes in which, eg., periodic boundary conditions are applied to a finite subset of an ostensibly unbounded domain. Ralf was able to push through the use of wavelets to probe and partially answer these questions, and tell a fairly convincing story, in the special case of the KS equation. In doing so, he reviewed a vast literature: his thesis remains one of the best introductions to spatiotemporal `chaos.' More significantly, he was able to develop one of the first well-founded classes of local models by appropriately periodizing the subdomain and replacing the large scales by stochastic forcing.

At the time of his graduation I ranked Ralf in the top quarter of my 22 PhD students (at Cornell and Princeton): the equal of Andrew Szeri (1988, ONR Fellowship, currently Assoc Prof in ME, UC Berkeley) or Pieter Swart (1990, currently staff member, Los Alamos NL) at similar stages in their careers. Three years later, I remain equally impressed, although it has taken some time for him to find his feet. He has taken his teaching at Michigan very seriously, and is, by all accounts, a great success (he gives excellent research talks, and writes extremely well). This slowed his research output for a couple of years, but he is now moving rapidly again.

Specifically, he has extended the tools he developed, following Collet et al., to prove analyticity and attractor dimension bounds for `extended' KS, to a broader class of PDEs. He has pushed his studies of stochastic dynamics of large scales on to a higher order problem - the Nikolaevskii model - in which scale separation is clearer (work with D.Cai). He is also working with C. Doering on rigorous bounds for bulk transport and dissipation rates in turbulent flows. The emphasis in all this work is to develop analytical tools leading to rigorous (if conservative) bounds, and to use careful numerical simulation to probe mechanisms and thereby improve the analysis and understanding of extended systems. He has chosen a difficult area, but he has the knowledge, creativity, independence and drive to make a real impact. Indeed, he is already doing so. I strongly support his application; he would make a wonderful colleague.

Please call me at 609-258-2958 if you wish to discuss his application further.

Sincerely yours,

Philip J. Holmes
Professor of Mechanics and Applied Mathematics

PJH/vm

UNIVERSITY OF MICHIGAN

Department of Mathematics
Ann Arbor, MI 48109-1109, USA

(734)-763-3505
fax: (734)-763-0937
krasny@math.lsa.umich.edu

December 16, 2001

Letter of Recommendation for **Ralf Wittenberg**

I'm writing to recommend Ralf Wittenberg for a tenure-track position in applied mathematics. Ralf has been at Michigan since 1999 after spending a year as a postdoc at the Institute for Mathematics and its Applications in Minnesota. Ralf has expertise in applied analysis, dynamical systems, and numerical simulations. Much of his research deals with the Kuramoto-Sivashinsky (KS) equation, a canonical partial differential equation that is relevant in many areas of fluid dynamics and physics. The solution of the KS equation undergoes a transition to chaos for a certain choice of parameter values and this process is of great interest. Ralf is working to develop a low-dimensional model that captures the dynamics of the full KS equation. His approach uses a wavelet decomposition to investigate the transfer of energy between modes that are separated in location and scale. The work is quite important since there is hope that the insights gained from the KS equation can be applied to the problem of turbulence in the Navier-Stokes equations. This is a continuation of Ralf's thesis work with Phil Holmes at Princeton. Another line of related work deals with a higher-order generalization of the KS equation, in collaboration with David Cai and David McLaughlin at Courant. Ralf is also working here at Michigan with Charlie Doering on a different problem, developing rigorous bounds for heat transport in turbulent convection. These topics are outside my own field of research and I'll let the experts describe his accomplishments in more detail. In the remainder of this letter I want to describe Ralf's other scholarly activities during his time at Michigan, starting with his teaching.

Simply put, Ralf did a superb job here in teaching. His teaching load was two courses per semester, although we did manage to reduce it to one course in the Winter 2001 semester. I had close interaction with Ralf in my capacity as coordinator of applied honors calculus (Math 156). Ralf taught the course twice, in 1999 and 2001. Each instructor is in charge of a small section of 30 students or less. Ralf showed great dedication to the course and the students, e.g. writing up homework solutions and holding extra review sessions before exams. I met with Ralf and the other instructors each week to discuss the direction of the course. His comments on these occasions were invariably helpful and he often liked to discuss the course with me outside the scheduled instructors' meetings. I attended one of his classes and observed that his lecturing style was clear and well-organized, and at the right level for the students.

Ralf's work in Math 156 is just the beginning because he taught a variety of other courses here as well; differential equations for sophomores (Math 216); applied partial differential equations (Math 454) and numerical analysis (Math 471) for juniors, seniors, and beginning graduate students; and partial differential equations for advanced graduate students (Math 656). I want to emphasize that Ralf volunteered to teach all these different courses. Many non-tenure-track instructors try to repeat the same courses each semester, but Ralf wouldn't be satisfied doing that. He saw that we needed instructors in these courses and this coincided with his own deep interest in teaching these topics. Professor Dick Canary, our Associate Chair for Education, will be writing about Ralf's teaching record from the Department's viewpoint, but I want to offer my own opinion that Ralf has been one of our most effective instructors in applied math courses.

Besides having a commitment to high-quality teaching, Ralf has excellent personal qualities and is an outstanding departmental citizen. He volunteered to organize the weekly applied math seminar and to enhance the seminar website, and he also volunteered to serve in the King/Chavez/Parks program which brings inner city middle-school students on campus to meet with University faculty.

In conclusion, it's a pleasure to give my strong recommendation to Ralf Wittenberg for a tenure-track position. I'm confident that he'll make valuable contributions in research, teaching, and service.

Sincerely yours,

Robert Krasny

Robert Krasny
Arthur F. Thurnau Professor
Professor of Mathematics

New York University
A private university in the public service
David W. McLaughlin, Director
Courant Institute of Mathematical Sciences
251 Mercer Street
New York, NY 10012–1185
Tel. (212) 998-3077
Fax (212) 995-4684
Telex 235128 NYU UR
Internet: dmac@cims.nyu.edu

December 20, 2001

Recommendation for **Ralf Wittenberg**:

I understand that Ralf Wittenberg is applying for an entry level position in your Department of Mathematics, and my purpose in this letter is to provide strong support for his candidacy.

Dr. Wittenberg works on chaos, and spatiotemporal chaos, in deterministic nonlinear wave equations. This is an extremely important area in applied mathematics today – as the interaction of stochasticity (either in deterministic or random systems) and nonlinearity is prevalent throughout nature. Dr. Wittenberg brings tools and methods of dynamical systems theory to his study of evolutionary partial differential equations. In addition to the tools from dynamical systems theory, Dr. Wittenberg also brings methods from wavelet analysis to these studies. This combination arms him with a strong set of analytical and numerical tools for these investigations, as exemplified in his excellent PhD dissertation "Local Dynamics and Spatiotemporal Chaos. The Kuramoto-Sivashinsky Equation: A Case Study", with Professor Philip Holmes as his advisor.

In addition to this work, he has performed a careful and controlled study of the use of "normal form methods" for specific partial differential equations – carefully documenting their (limited) usefulness for pde's. Recently, he has begun a series of studies concerning the characterization of spatiotemporal chaos, low dimensional chaos within pde setting, and effective stochastic dynamics. The goal of the latter is to construct and validate effective equations for the description of large scale, long time behavior in spatiotemporal chaotic systems – descriptions which can be used for long time prediction within these systems.

His study of the sixth order Nikoloevskii equation (with David Cai) is particularly promising in that this equation possesses a clear scale separation permitting very detailed and precise information about the validity of the effective equations. Moreover, the effective diffusion coefficient for this model possesses its own distinct characteristics, which predict distinctive phenomena in the behavior of the nonlinear wave.

In addition to his work in spatiotemporal chaos for pde's, Dr. Wittenberg has begun working on bounds for bulk flow quantities in fluid dynamics. This work was initiated during his instructorship at the University of Michigan, jointly with Charlie Doering and others. I am certain that other referees will address it specifically.

Dr. Ralf Wittenberg is an exceptionally clear expositor, both in writing and lectures. He is a very conscientious person and careful scholar. I am certain that he is an outstanding teacher and would make an excellent colleague. He has the potential to become a fine researcher, and I recommend him to you with enthusiasm.

Sincerely yours,

D W McLaughlin

David W. McLaughlin

Academic Employment in Mathematics

AMS Standard Cover Sheet

This cover sheet is provided as an aid to departments in processing job applications. It should be included with your other application material. Please print or type. Do not send this form to the AMS.

Last (Family) Name: *Wittenberg*

First Name or Initial: *Ralf*

Middle Name or Initial: *Werner*

Address through next June:

Department of Mathematics, University of Michigan
2072 East Hall, 525 E. University Ave.
Ann Arbor, MI 48109-1109

Home Phone: *(734) 327-9672*

e-mail address: `ralf@math.lsa.umich.edu`

Current Institutional Affiliation:

Department of Mathematics, University of Michigan

Work Phone: *(734) 763-5725*

Highest Degree held or expected *Ph.D.*

Granting Institution *Princeton University* Date (optional) *1998*

Ph.D. Advisor: *Philip J. Holmes*

Ph.D. Thesis Title (optional) *Local Dynamics and Spatiotemporal Chaos...*

Indicate the mathematical subject areas in which you have done research using, if applicable, the Mathematics Subject Classification. If listing more than one number, list first the one number which best describes your current primary interest.

Primary Interest *37*

Secondary Interests (optional) *35, 34, 76, 92*

Give a very brief synopsis of your current research interests in the box below (e.g. finite group actions on four-manifolds). Avoid special mathematical symbols.

Dynamical systems; complex spatial and temporal dynamics in extended systems, spatiotemporal chaos; pattern formation; nonlinear evolution equations; low-dimensional models; differential equations; applied mathematics; fluid dynamics; mathematical biology

Most recent position held, if any, post Ph.D.

University or Company *University of Michigan*

Position Title *Assistant Professor* Dates *Fall 1999-Summer 2002*

Indicate the position for which you are applying and position posting code, if applicable

Postdoctoral position

If unsuccessful for this position, would you like to be considered for a temporary position?

[X] Yes [ ] No If yes, please check the appropriate boxes.

[X] Postdoctoral Position [X] 2+ Year Position [X] 1 Year Position

List the names and affiliations of up to four individuals who will provide letters of recommendation if asked.

Mark the box provided for each individual whom you have already asked to send a letter.

[X] 1. *Philip Holmes, Princeton University; pholmes@rimbaud.princeton.edu*

[X] 2. *David McLaughlin, Courant Institute, NYU; dmac@cims.nyu.edu*

[X] 3. *Charles Doering, University of Michigan; doering@math.lsa.umich.edu*

[X] 4. *Robert Krasny, University of Michigan; krasny@math.lsa.umich.edu*

RALF W. WITTENBERG

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Citizenship: South African

RESEARCH INTERESTS

Complex spatial and temporal dynamics in extended systems, spatiotemporal chaos; dynamical systems; pattern formation; nonlinear evolution equations; differential equations; applied mathematics; fluid dynamics, mathematical biology.

WORK EXPERIENCE

Assistant Professor, Sep. 1999 – Aug. 2002,
Department of Mathematics, University of Michigan, Ann Arbor, MI.

Postdoctoral Member, Sep. 1998 – Aug. 1999,
Mathematics in Biology program year,
Institute for Mathematics and its Applications, University of Minnesota, Minneapolis, MN.

EDUCATION

Sep. 1993 – Aug. 1998: Ph.D.: Applied and Computational Mathematics.
Princeton University, Princeton, NJ. Advisor: Philip J. Holmes.
Dissertation title:
Local Dynamics and Spatiotemporal Chaos. The Kuramoto-Sivashinsky Equation: A Case Study
M.A., Applied and Computational Mathematics, May 1995.

Feb. 1992 – Aug. 1993: Master of Science (with distinction): Applied Mathematics.
University of Cape Town, South Africa. Advisor: George F.R. Ellis.
Dissertation title: *Models of Self-Organization in Biological Development*

Feb. 1991 – Nov. 1991: Bachelor of Science (Honours) (*summa cum laude*): Physics.
University of Natal, Durban, South Africa. Advisor: Manfred A. Hellberg.

Feb. 1988 – Nov. 1990: Bachelor of Science (*cum laude*): Mathematics, Physics, Chemistry.
University of Natal, Durban, South Africa.

PUBLICATIONS

Journal Articles

1. "Large-scale contributions to spatiotemporal chaos." In preparation.

2. "Bounds on the Nusselt-Rayleigh number relationship in a fluid layer with a constant heat flux." In preparation, with J. Otero, C. R. Doering, R. A. Worthing, H. Johnston and B. J. Keen.

3. Ralf W. Wittenberg, "Dissipativity, analyticity and viscous shocks in the (de)stabilized Kuramoto-Sivashinsky equation." To appear in *Physics Letters A*.

4. Ralf W. Wittenberg and Philip Holmes, "Spatially localized models of extended systems." *Nonlinear Dynamics*, **25** (1/3), 111–132 (2001).

5. Ralf W. Wittenberg and Philip Holmes, "Scale and space localization in the Kuramoto-Sivashinsky equation." *Chaos*, **9** (2), 452–465 (1999).

6. Philip J. Holmes, John L. Lumley, Gal Berkooz, Jonathan C. Mattingly and Ralf W. Wittenberg, "Low-dimensional models of coherent structures in turbulence." *Physics Reports*, **287** (4), 337–384 (1997).

7. Ralf W. Wittenberg and Philip Holmes, "The limited effectiveness of normal forms: a critical review and extension of local bifurcation studies of the Brusselator PDE." *Physica D*, **100**, 1–40 (1997).

Conference Proceedings and other Publications

1. "Kuramoto-Sivashinsky equation." Invited article, to appear in *Encyclopaedia of Mathematics*, Supplement III, Kluwer Academic Publishers.

2. Ralf W. Wittenberg, "Local Dynamics and Spatiotemporal Chaos. The Kuramoto-Sivashinsky Equation: A Case Study." Ph.D. thesis, Princeton University, 1998.

3. Philip J. Holmes, Jonathan C. Mattingly and Ralf W. Wittenberg, "Low-Dimensional Models of Turbulence or, the Dynamics of Coherent Structures." In *From Finite to Infinite Dimensional Dynamical Systems* (J.C. Robinson and P.A. Glendinning, eds.), NATO Science Series II, vol. 19 (Proceedings of the NATO Advanced Study Institute, Cambridge, UK, 21 August–1 September 1995), Kluwer Academic Publishers, Dordrecht, 2001, pp. 177–215.

4. Ralf W. Wittenberg, "Models of Self-Organization in Biological Development." M.Sc. thesis, University of Cape Town, 1993.

5. R.W. Wittenberg, M.A. Hellberg and W. Feneberg, "Ambipolar Transport in a Magnetically Perturbed Tokamak Edge-Region." Proceedings, International Conference on Plasma Physics, Innsbruck (1992).

PRESENTATIONS

Invited Talks

Nov. 2001: Applied Mathematics Seminar, University of Michigan, Ann Arbor, MI.

Oct. 2001: Nonlinear Science Seminar, Northwestern University, Evanston, IL.

Sep. 2001: Partial Differential Equations Seminar, Indiana University, Bloomington, IN.

May 2001: SIAM Conference on Applications of Dynamical Systems, Snowbird, UT.

Dec. 2000: Joint Physics Seminar, Bowling Green State University/University of Toledo, Bowling Green, OH.

May 1999: Minisymposium "Characterization of Spatiotemporal Chaos", SIAM Conference on Applications of Dynamical Systems, Snowbird, UT.

Jan. 1999: Applied Mathematics Seminar, University of Michigan, Ann Arbor, MI.

Dec. 1998: Postdoctoral Seminar, IMA, University of Minnesota, Minneapolis, MN.

Oct. 1997: Cornell Workshop on POD-Galerkin Models for the Dynamics and Control of Complex Flows, Ithaca, NY.

June 1997: Joint South African Mathematical Society/AMS Conference, Pretoria.

May 1997: SIAM Conference on Applications of Dynamical Systems, Snowbird, UT.

Feb. 1996: Nonlinear Science Seminar, Princeton University, Princeton, NJ.

Posters

Sep. 2000: Fluid Dynamics: Theory, Computation & Application, Michigan Interdisciplinary Mathematics Meeting III, Ann Arbor, MI.

Jun. 2000: Nonlinear Analysis 2000 $\rightarrow$, Courant Institute, New York, NY.

TEACHING EXPERIENCE

University of Michigan, Assistant Professor, 1999–2002.
Applied Honors Calculus II: Integration (*Math 156*, Fall 1999: 2 sections, Fall 2001);
Differential Equations (*Math 216*, Winter 2000: 2 sections, Summer 2000, Spring 2001);
Applied Honors Calculus IV: Differential Equations (*Math 256*, Fall 2000: 2 sections);
Partial Differential Equations (*Math 656*, Winter 2001, graduate level);
Boundary Value Problems for Partial Differential Equations (*Math 454*, Fall 2001, Winter 2002);
Introduction to Numerical Methods (*Math 471*, Winter 2002).

Princeton University, Teaching Assistant, 1994–1996.
Graduate course in asymptotic analysis (3 times), undergraduate course in ODEs.

University of Cape Town, Teaching Assistant, 1992–1993.
Ordinary Differential Equations (1993): Lecturer;
Advanced Calculus, Mechanics, ODEs (1992): Teaching assistant.

University of Natal, Durban, Teaching Assistant, 1991.
Supervision of undergraduate physics laboratories, and grading of laboratory reports.

ACADEMIC HONORS

Princeton University (1993–98):
Charlotte Elizabeth Procter Honorific Fellowship
Princeton University First-Year Fellowship, and Entering Prize of $2500
University of Cape Town Queen Victoria Scholarship

University of Cape Town (1992–93):
AECI Postgraduate Research Fellowship
UCT Research Associateship

University of Natal, Durban (1988–91):
University of Natal Postgraduate Scholarship
AECI Undergraduate Scholarship
Various university academic merit awards

CONFERENCES, WORKSHOPS, AND SUMMER SCHOOLS ATTENDED

May 2001: *SIAM Conference on Applications of Dynamical Systems*, Snowbird, UT.

Nov. 2000: *Midwest Partial Differential Equations Seminar*, University of Chicago, Chicago, IL.

Sep. 2000: *Fluid Dynamics: Theory, Computation & Application*, Michigan Interdisciplinary Mathematics Meeting III, Ann Arbor, MI.

Jun. 2000: *Nonlinear Analysis 2000* →, Courant Institute, New York, NY.

May 1999: *SIAM Conference on Applications of Dynamical Systems*, Snowbird, UT.

Sep. 1998 – Jun. 1999: *Mathematics in Biology*, Theme Year
Institute for Mathematics and its Applications, University of Minnesota, Minneapolis, MN.

May 1998: *Pattern Formation in Continuous and Coupled Systems*, IMA Workshop, Minneapolis, MN.

Oct. 1997: *Cornell Workshop on POD-Galerkin Models for the Dynamics and Control of Complex Flows*, Ithaca, NY.

Jun. 1997: *Joint Meeting*, South African Mathematical Society/AMS/SAMSA, Pretoria, South Africa.

May 1997: *SIAM Conference on Applications of Dynamical Systems*, Snowbird, UT.

Jun. – Jul. 1996: *Probability*, Summer School, Institute for Advanced Study/Park City Mathematics Institute, Princeton, NJ.

Aug. 1995: *From Finite to Infinite Dimensional Dynamical Systems*, Summer School, NATO Advanced Study Institute, Newton Institute for Mathematical Sciences, Cambridge, UK.

Jan. 1995: *Dynamics Days*, Houston, TX.

Jan. – Feb. 1993: *Dynamical Systems and Nonlinear Analysis*, Summer School, Cape Town, South Africa.

Jan. 1992: *Chaos and Quantum Chaos*, Summer School, Blydepoort, South Africa.

OTHER ACADEMIC ACTIVITIES

Fall 2001: Host for middle school students in King/Chavez/Parks visitation program.

Sep. 2001 – Apr. 2002: Organizer, Applied and Interdisciplinary Mathematics seminar, University of Michigan.

Jan. 1999 – May 1999: Co-organizer, postdoctoral seminar, IMA, University of Minnesota.

Sep. 1994 – Jun. 1998: Founder and co-organizer, Applied Mathematics graduate student seminar, Princeton University.

Journal Referee: (17 articles):
Physica D: Nonlinear Phenomena; *Nonlinearity*; *Philosophical Transactions of the Royal Society of London A*; *Journal of Physics A: Mathematical and General*; *Journal of Physics D: Applied Physics*; *SIAM Journal on Applied Mathematics*; *Physics Letters A*; *Journal of Mathematical Analysis and Applications.*

Professional Memberships: SIAM, AMS.

REFERENCES

- *Philip J. Holmes*
  Program in Applied and Computational Mathematics and Department of Mechanical and Aerospace Engineering, Princeton University
  `pholmes@rimbaud.princeton.edu`
- *David W. McLaughlin*
  Courant Institute of Mathematical Sciences, New York University
  `dmac@cims.nyu.edu`
- *Charles R. Doering*
  Department of Mathematics, University of Michigan
  `doering@math.lsa.umich.edu`
- *Robert Krasny*
  Department of Mathematics, University of Michigan
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Research Statement — RALF W. WITTENBERG

My research interests fall in the general field of nonlinear dynamics, particularly complex spatial and temporal dynamics and pattern formation, with a view also towards applications in various physical, chemical or biological contexts. My current interests focus mainly on spatiotemporally complex and chaotic systems, typically spatially extended infinite-dimensional dynamical systems modeled by partial differential equations (PDEs).

As I outline below, I seek to combine a range of techniques, including methods from finite-dimensional dynamical systems and bifurcation theory, low-dimensional models, applied analysis of the PDEs, asymptotic analysis and careful scientific computation, to investigate the properties and origins of the observed spatiotemporal complexity. The work described below includes my main current research directions and achievements. I am also interested in problems arising in other disciplines, however, and would in future especially like to increase my focus on dynamics and pattern formation in biological applications; my past experience in this area includes my participation in the IMA Program Year on Mathematical Biology and my master's thesis, which concerned models in biological development.

Local Bifurcation and Normal Form Theory An early triumph of the use of center manifold and normal form reduction and bifurcation theory to study the spatiotemporally complex dynamics of a high-dimensional dynamical system near a bifurcation point was the prediction by Guckenheimer [19], of Šil'nikov chaos in the Brusselator reaction-diffusion system by an analysis of the unfolding of the codimension two transcritical (Turing)/Hopf bifurcation. This work stimulated many detailed studies of normal forms and the complex dynamics occurring in the unfolding of this and other bifurcations of codimension greater than one. However, through careful comparison of the predictions of normal form theory with approximations of the full PDE (using *inter alia* computer algebra and numerical bifurcation calculations), I found that this normal form approach has limited usefulness for the understanding of spatiotemporal complexity for the Brusselator PDE, in that the parameter ranges in which the normal form predictions may be applied, are very small [42].

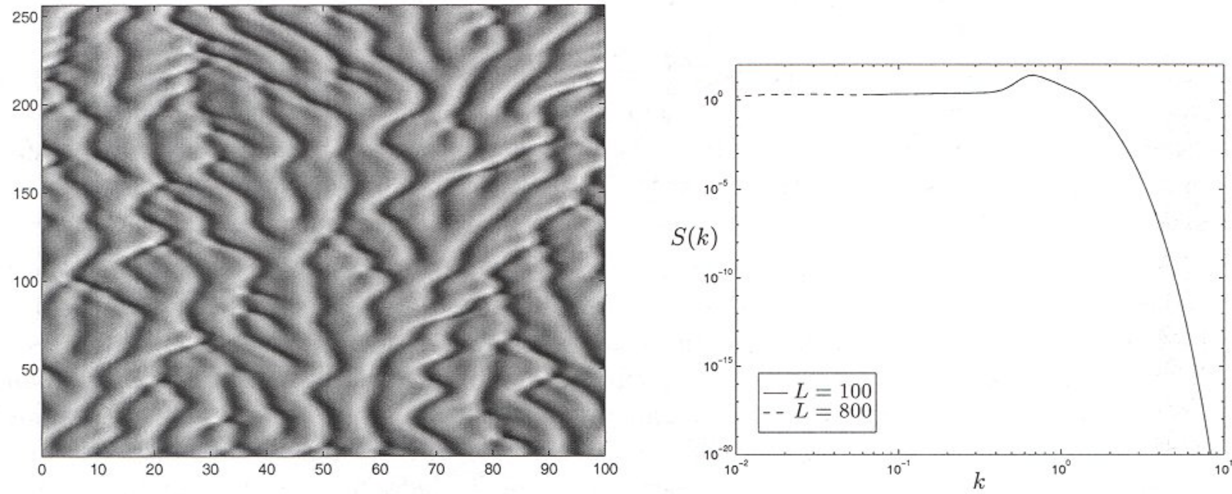
Spatiotemporal Chaos and the Kuramoto-Sivashinsky equation

More recently I have largely concentrated on studying systems with much more complex spatiotemporal dynamics [12]. Spatiotemporal chaos (STC) is a fascinating and incompletely understood phenomenon. I have developed new approaches and used existing analytical, numerical and modeling tools for the investigation and characterization of STC, as I outline below, and I hope to continue to contribute to the understanding of STC.

I have focused particularly on a class of equations including the one-dimensional Kuramoto-Sivashinsky (KS) equation,

$$u_t + u_{xxxx} + u_{xx} + uu_x = 0 \qquad (1)$$

typically with periodic boundary conditions on a domain of length L. This equation (see [40] for an overview) has arisen in many contexts in which a long-wavelength primary instability is coupled to small-scale damping in the presence of appropriate symmetries [31], such as plasma ion mode instabilities, chemical phase turbulence, and flame front instabilities. The number of linearly unstable Fourier modes is proportional to L, and for increasing L a rich bifurcation sequence through cellular states, standing, traveling and modulated traveling waves and heteroclinic cycles and more complex



(a) (b)

Figure 1: (a) Gray-scale view (peaks are light, troughs are dark) of a solution of the KS equation (1) on the spatiotemporally chaotic attractor for $L = 100$, covering 256 time units separated by $\Delta t = 1$, clearly showing the typical cellular structure, travelling cells, and creation and annihilation of peaks. (b) Rescaled Fourier power spectrum $S(k) = L\langle|\hat{u}_k|^2\rangle$, for $L = 100$ and $L = 800$.

spatiotemporal solution phenomena has been described [24, 27], which eventually all appear to become unstable to a spatiotemporally chaotic attractor; Fig. 1(a) shows a typical evolution.

Characterization of Spatiotemporal Chaos In my thesis [39], I showed by extensive numerical simulations that a projection onto a suitable (spline) wavelet basis can supplement real and Fourier space statistics [34] (see Fig. 1(b)) to elucidate features of the dynamics on the attractor. The wavelet approach (following [2, 17]) effectively separates and distinguishes between scales having qualitatively different wavelet coefficient distributions, invariant in the STC regime: there are large scales of slow Gaussian fluctuations, "active" scales near the peak of the power spectrum involving spatially localized interactions of energetic coherent structures, and strongly damped small scales with exponentially decaying energy.

Furthermore, I showed ([39], summarized in [43, 44]) how novel simulations help one to infer the existence of a characteristic spatial interaction length l_c for instantaneous dynamical influences. Other numerical experiments in which one eliminates or forces different wavelet levels provide a detailed picture of the dynamical contributions of the different wavelet levels to the overall spatiotemporally complex dynamics. In computations in progress, I am able to use the wavelet analysis to observe the localized nature of the energy transfer cascade in both Fourier and physical space, extending the independent recent related work of [32]. I hope to develop these wavelet-based methods for studying the dynamics in space and scale to other problems (see also [46]), including higher-dimensional systems.

An important consequence of my (continuing) work, deduced through the wavelet-based numerical experiments and by comparison with related models with modified dispersion relations (discussed below), has been to clarify the role of the large scales in the flat region of the spectrum (Fig. 1(b)) [38]: though they contain a fairly small fraction of the total energy, if they are absent or suppressed the dynamics decay to a steady roll state, while in the presence of excessive large-scale driving the STC gives way to sustained Burgers shock-like solutions. Remarkably, though, experiments in which the large-scale modes are replaced by uncorrelated, autonomously generated stochastic Langevin (Ornstein-Uhlenbeck) processes display active-scale dynamics and statistics quite similar to those typical of the full (deterministic) PDE. That is, STC appears to be driven and maintained by a "heat bath" of large-scale random excitation at (self-consistently) appropriate amplitudes. Conversely, as discussed further below the active-scale chaos induces this effective stochasticity at the large scales. Together, these two ideas account for STC via the mutual interactions between the active and large scales.

Low-Dimensional Models Guided by our results on scale and space localization, following [2, 13, 17] we have investigated spatially localized models for the KS equation. The approach of obtaining low-dimensional models capturing the essential features of high-dimensional systems (reviewed in [21]) typically involves projection of the governing equations onto suitable modes obtained via the proper orthogonal (POD, or Karhunen-Loève) decomposition. However, the POD modes in our translation-invariant system are Fourier modes, which are not spatially localized, so in my thesis I studied several models for the evolution of a subset of the full wavelet basis, the neglected modes modeled by external forcing.

To summarize the results: diverse numerical experiments indicate that KS dynamics and statistics typical of the STC regime are captured only in models in which translational symmetry is maintained, and that there is persuasive evidence that short periodized systems, internally forced at their largest scales, may form minimal models for chaotic dynamics in arbitrarily large domains [44]. This suggests that my planned detailed study of the dynamics and bifurcations of a relatively low-dimensional model of KS dynamics subject to large-scale forcing may be promising. The successful construction of such a low-dimensional system modeling the active scales and displaying features of extensive chaos would give a more precise meaning to the idea that an extended chaotic system in the "thermodynamic limit" can be understood as a collection of weakly interacting small subsystems [12, 18] by *explicit construction* of such minimal subsystems.

Generalizations of the KS equation

Consider the family of self-adjoint higher-order generalizations of the KS equation,

$$\partial_t u = \left(-\partial_x^2\right)^p \left[\varepsilon^2 u - \left(1+\partial_x^2\right)^2 u\right] - u\partial_x u, \qquad x \in [0,L], \quad \text{periodic boundary conditions} \tag{2}$$

where $p \geq 0$ is an integer. For $p = 0$, $\varepsilon^2 = 1$ this is a rescaled version of the KS equation (1), and in general for $p = 0$ the above equation describes the linearly damped KS equation [31], for which it has been argued that the route to STC occurs via spatiotemporal intermittency as ε^2 increases towards 1 [7, 16]. For $p = 1$, this equation (proposed for the propagation of longitudinal seismic waves in viscoelastic media and sometimes known as the Nikolaevskii model [1, 45]) has attracted recent attention [30, 37] as a model for pattern formation in the presence of Galilean symmetry. The neutrally stable long-wave mode coupled to the KS-like short-wave instability drives the dynamics so that (for sufficiently large L) by contrast with the KS case $p = 0$ there is a direct transition from a spatially uniform state to spatiotemporal chaos as ε increases above zero [36]. In view of this novel behavior, it is of interest to study spatiotemporally chaotic phenomena in the Nikolaevskii model and its generalizations (2), as they may also shed light on the KS equation and on STC in general.

Analysis of the PDEs and Bounds on the Attractor Following the Fourier space methods of Collet *et al.* [8, 9], I have recently proved the dissipativity in L^2 and analyticity of solutions of (2). For $\varepsilon^2 \leq 1$, $p \geq 0$, the radius of an L^2 absorbing ball has a bound of the form $\limsup_{t\to\infty} \|u\|_2^2 \leq K_p \varepsilon^{(12p+26)/(2p+5)} L^{(8p+16)/(2p+5)}$, which extends the results for the damped KS equation ($p = 0$) reported in [39, 41]. I plan to continue the study of mathematical properties such as attractors and inertial manifolds [35] for the model class of systems (2), as it is as yet in general unknown whether and how the properties of dissipative dynamical systems depend on the order of the differential operator.

Similar computations for $p = 0$ and $\varepsilon^2 \geq 1$ [41] shed light on an open problem concerning the KS thermodynamic limit $L \to \infty$: Numerical evidence strongly indicates that for large L one has "extensive chaos" [18], in which due to rapid decay of spatial correlations [43] local dynamics and pointwise bounds on solutions are asymptotically independent of system size, while extensive quantities such as the energy $\|u\|_2^2$ and the number of positive Lyapunov exponents [29] should be proportional to L. However, the available bounds for the KS equation (see [41]) all appear to be suboptimal; for instance, the best rigorous estimate of the L^2 absorbing ball is $\|u\|_2^2 \leq KL^{16/5}$.

For $p = 0$ I have shown [39, 41] that an L^2 bound of the above form holds also for the KS equation with $\varepsilon^2 \geq 1$, in which case the linear term is *destabilizing*, driving energy into the large scales. However, for sufficiently large ε^2 ($\varepsilon^2 > 1.5$ seems to suffice) numerical solutions indicate that the dynamics are attracted to an internal layer shock-like solution, whose existence is corroborated by a large-ε asymptotic analysis. The energy for this solution scales as $\|u\|_2^2 \sim K\varepsilon^4 L^3$, consistent with the rigorous bound but not extensive [41]. This example shows that since the previous analytical methods also work for $\varepsilon^2 > 1$, for which extensivity fails, a successful proof of extensivity for the KS equation will require an approach that *fails* on this $p = 0$, $\varepsilon^2 > 1$ problem, possibly via a careful analysis of large-scale modal interactions. Detailed asymptotic analysis and further investigations of the stability and bifurcations of this shock-like solution are continuing, in the hope that they might further clarify the KS limit $\varepsilon^2 \to 1$.

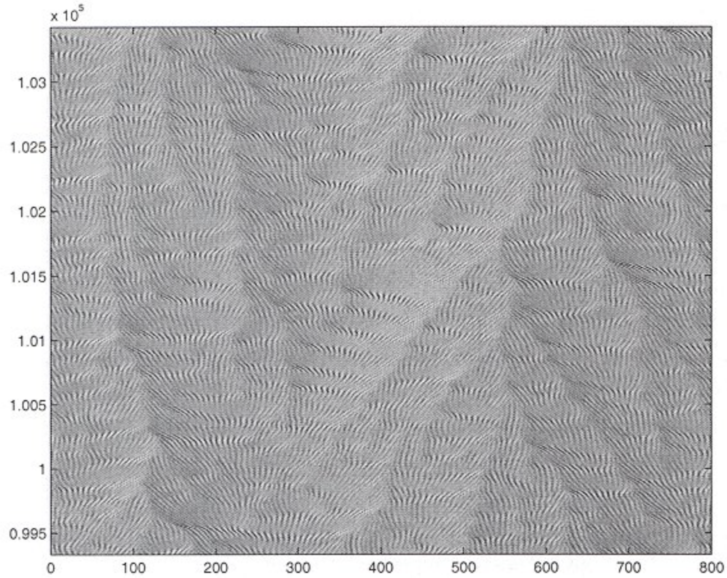
Effective Stochastic Dynamics at the Large Scales There has been considerable effort devoted towards understanding the effective Gaussian dynamics at large length and time scales (see [4, 28, 43]). Much of the effort has concentrated on validating via renormalization group and other approaches the conjecture of Yakhot [47] that the coarse-grained large-scale dynamics may be well-described by a noise-driven Burgers (or Kardar-Parisi-Zhang) equation

$$u_t = \nu u_{xx} + \lambda u u_x + f,$$

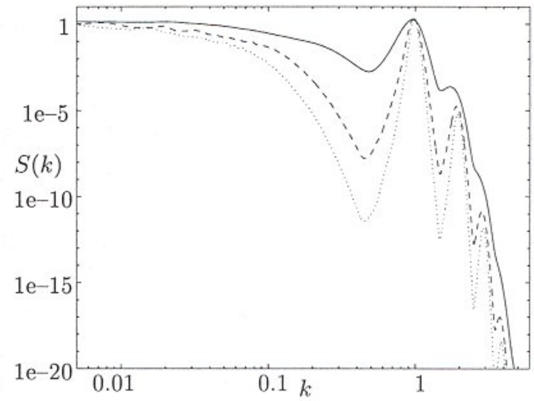
with f a stochastic force, the (deterministic) chaotic dynamics at active and small scales forcing the large scales and renormalizing the diffusion coefficient to generate a positive effective ν.

A constructive approach to extracting the effective slow-mode, large-scale dynamics was initiated by Zaleski [48] (see also [5]) who proposed an explicit procedure to eliminate wavelengths shorter than some cutoff. The effective viscosity ν ($\nu \sim 10$ for the KS equation [20, 48]) could then be self-consistently estimated from time-dependent correlations. However, the applicability of this procedure for the KS equation is unclear, as there is no good separation of scales or obvious choice of cutoff wavelength (see Fig. 1(b)). Thus (jointly with David Cai) I am currently investigating this procedure for the 6th-order Nikolaevskii model ($p = 1$ in (2)), for which extensive computations show, as in Fig. 2(a), that for large L and small ε there is a clear separation of scales; as seen clearly in Fig. 2(b), near $k \approx 0.5$ there is a range of modes increasingly damped for decreasing ε, so that a cutoff wavelength in this range may be chosen much more plausibly than for the KS model. Preliminary computations indicate that Zaleski's constructive procedure appears to yield an effective diffusion coefficient ν near zero for small ε, implying that the large-scale dynamics may in the limit be described by an *inviscid* Burgers equation.

The goal of our continuing research is to understand systematically the mode elimination and coarse-graining procedure, in the hope that this may partially validate the application of stochastic methods to deterministic problems [28], and shed light on the fundamental question: How can stochasticity arise from fully deterministic dynamics?



(a)



(b)

Figure 2: (a) A solution of the generalized KS equation (2) ($p = 1$: Nikolaevskii model) for $L = 800$ and $\varepsilon^2 = 0.04$; compare Fig. 1(a), noting the change in length scales. (b) Power spectrum for solutions of (2) ($p = 1$) for $L = 1600$ and $\varepsilon^2 = 0.04$ (solid), $\varepsilon^2 = 0.01$ (dashed) and $\varepsilon^2 = 0.005$ (dotted); for small ε the longest-wave (low k) Fourier modes are increasingly well separated from the active-scale linearly unstable modes near $k = 1$ (compare Fig. 1(b)).

Fluid Dynamics

Bounds on Bulk Flow Quantities: Convection A fundamental question in the Rayleigh-Bénard problem of heating a bounded fluid layer from below is to estimate the total rate of bulk heat transport directly from the governing Boussinesq equations, specifically, in bounding the Nusselt number Nu (the non-dimensionalized measure of convective heat transfer) in terms of the Rayleigh number Ra, defined as the non-dimensionalized (averaged) temperature difference across the plates. In the high-Ra convective turbulence limit, the Nu–Ra relationship is expected to follow a scaling law, $Nu \sim Ra^\gamma$. While experiments and heuristic physical scaling laws suggest bounds with exponents roughly between 1/4 and 1/3 (current values lie in the range $\gamma \approx 0.28 - 0.31$; see [25] and references therein), the best available rigorous bounds (without additional smoothness assumptions [10, 26] or constraints such as infinite Prandtl number [11]) yield an exponent of $\gamma = 1/2$ [15].

In seeking to account for this discrepancy with experiment, we have been examining more carefully the assumptions of this model, particularly the temperature boundary conditions. In practice, the experimental fluid is bounded by plates of finite thickness and conductivity, and especially for high temperature gradients (large Ra) the usual assumption of a fixed (Dirichlet) uniform temperature distribution on the fluid boundaries appears unrealistic. We have thus studied the Neumann (fixed flux) problem [6], and have formulated appropriate variational principles and extracted upper bounds in both the Howard-Busse [3, 22] and Doering-Constantin [14, 15] formulations [33]. Currently we are studying the more physically realistic problem, in which the finite thickness d and conductivity κ of the plates (modeled by a heat equation [23]) are taken into account, and I have recently demonstrated that while the rigorous bound still has a scaling exponent of 1/2, the fixed temperature problem, corresponding to the limits $d \to 0$ and/or $\kappa \to \infty$, seems to be a singular limit of the full problem.

References

[1] I. A. Beresnev and V. N. Nikolaevskiy. A model for nonlinear seismic-waves in a medium with instability. *Physica D*, 66(1–2):1–6, 1993.

[2] G. Berkooz, J. Elezgaray, and P. Holmes. Coherent structures in random media and wavelets. *Physica D*, 61:47–58, 1992.

[3] F. H. Busse. On Howard's upper bound for heat transport by turbulent convection. *J. Fluid Mech.*, 37:457–477, 1969.

[4] D. Cai and D. W. McLaughlin. Chaotic and turbulent behavior of unstable one-dimensional nonlinear dispersive waves. *J. Math. Phys.*, 41(6):4125–4153, 2000.

[5] D. Cai, D. W. McLaughlin, and J. Shatah. Spatiotemporal chaos and effective stochastic dynamics for a near-integrable nonlinear system. *Phys. Lett. A*, 253(5–6):280–286, 1999.

[6] C. J. Chapman and M. R. E. Proctor. Nonlinear Rayleigh-Bénard convection between poorly conducting boundaries. *J. Fluid Mech.*, 101(4):759–782, 1980.

[7] H. Chaté and P. Manneville. Transition to turbulence via spatiotemporal intermittency. *Phys. Rev. Lett.*, 58(2):112–115, 1987.

[8] P. Collet, J.-P. Eckmann, H. Epstein, and J. Stubbe. A global attracting set for the Kuramoto-Sivashinsky equation. *Commun. Math. Phys.*, 152:203–214, 1993.

[9] P. Collet, J.-P. Eckmann, H. Epstein, and J. Stubbe. Analyticity for the Kuramoto-Sivashinsky equation. *Physica D*, 67:321–326, 1993.

[10] P. Constantin and C. R. Doering. Heat transfer in convective turbulence. *Nonlinearity*, 9:1049–1060, 1996.

[11] P. Constantin and C. R. Doering. Infinite Prandtl number convection. *J. Stat. Phys.*, 94(1/2):159–172, 1999.

[12] M. Cross and P. Hohenberg. Pattern formation outside of equilibrium. *Rev. Mod. Phys.*, 65(3):851–1112, 1993.

[13] H. Dankowicz, P. Holmes, G. Berkooz, and J. Elezgaray. Local models of spatio-temporally complex fields. *Physica D*, 90:387–407, 1996.

[14] C. R. Doering and P. Constantin. Energy dissipation in shear driven turbulence. *Phys. Rev. Lett.*, 69(11):1648–1651, 1992.

[15] C. R. Doering and P. Constantin. Variational bounds on energy dissipation in incompressible flows. III. Convection. *Phys. Rev. E*, 53(6):5957–5981, 1996.

[16] K. Elder, J. Gunton, and N. Goldenfeld. Transition to spatiotemporal chaos in the damped Kuramoto-Sivashinsky equation. *Phys. Rev. E*, 56(2):1631–1634, 1997.

[17] J. Elezgaray, G. Berkooz, and P. Holmes. Large-scale statistics of the Kuramoto-Sivashinsky equation: A wavelet-based approach. *Phys. Rev. E*, 54(1):224–230, 1996.

[18] H. S. Greenside. Spatiotemporal Chaos in Large Systems: How Complexity Scales with Size. In K. Coughlin, editor, *Semi-Analytic Methods for the Navier-Stokes Equations*, Proceedings of the CRM Workshop, Montréal, 1998. Centre de Recherches Mathématiques.

[19] J. Guckenheimer. On a codimension two bifurcation. In D. A. Rand and L.-S. Young, editors, *Dynamical Systems and Turbulence*, volume 898 of *Springer Lecture Notes in Mathematics*, pages 99–142. Springer-Verlag, New York, 1981.

[20] F. Hayot, C. Jayaprakash, and C. Josserand. Long-wavelength properties of the Kuramoto-Sivashinsky equation. *Phys. Rev. E*, 47(2):911–915, 1993.

[21] P. J. Holmes, J. L. Lumley, G. Berkooz, J. C. Mattingly, and R. W. Wittenberg. Low-dimensional models of coherent structures in turbulence. *Phys. Rep.*, 287(4):337–384, 1997.

[22] L. N. Howard. Heat transport by turbulent convection. *J. Fluid Mech.*, 17:405–432, 1963.

[23] D. T. J. Hurle, E. Jakeman, and E. R. Pike. On the solution of the Benard problem with boundaries of finite conductivity. *Proc. R. Soc. London A*, 296(1447):469–475, 1967.

[24] J. M. Hyman, B. Nicolaenko, and S. Zaleski. Order and complexity in the Kuramoto-Sivashinsky model of weakly turbulent interfaces. *Physica D*, 23:265–292, 1986.

[25] L. P. Kadanoff. Turbulent heat flow: Structures and scaling. *Physics Today*, pages 34–39, August 2001.

[26] R. R. Kerswell. New results in the variational approach to turbulent Boussinesq convection. *Phys. Fluids*, 13(1):192–209, 2001.

[27] I. G. Kevrekidis, B. Nicolaenko, and J. C. Scovel. Back in the saddle again: A computer assisted study of the Kuramoto-Sivashinsky equation. *SIAM J. Appl. Math.*, 50(3):760–790, 1990.

[28] A. J. Majda, I. Timofeyev, and E. Vanden Eijnden. Models for stochastic climate prediction. *Proc. Natl. Acad. Sci. USA*, 96(26):14687–14691, 1999.

[29] P. Manneville. Liapounov exponents for the Kuramoto-Sivashinsky model. In U. Frisch, J. Keller, G. Papanicolaou, and O. Pironneau, editors, *Macroscopic Modelling of Turbulent Flows*, volume 230 of *Lecture Notes in Physics*, pages 319–326, Berlin Heidelberg, 1985. Springer-Verlag.

[30] P. C. Matthews and S. M. Cox. One-dimensional pattern formation with Galilean invariance near a stationary bifurcation. *Phys. Rev. E*, 62(2):R1473–R1476, 2000.

[31] C. Misbah and A. Valance. Secondary instabilities in the stabilized Kuramoto-Sivashinsky equation. *Phys. Rev. E*, 49(1):166–183, 1994.

[32] H. Nakao, T. Mishiro, and M. Yamada. Visualization of correlation cascade in spatio-temporal chaos using wavelets. *Int. J. Bif. Chaos*, 11(5):1483–1493, 2001.

[33] J. Otero, C. R. Doering, R. A. Worthing, R. W. Wittenberg, H. Johnston, and B. J. Keen. Bounds on the Nusselt-Rayleigh number relationship in a fluid layer with a constant heat flux. In preparation.

[34] A. Pumir. Statistical properties of an equation describing fluid interfaces. *J. Physique*, 46:511–522, 1985.

[35] R. Temam. *Infinite-Dimensional Dynamical Systems in Mechanics and Physics*. Springer-Verlag, New York, 2nd edition, 1997.

[36] M. I. Tribelsky and K. Tsuboi. New scenario for transition to turbulence? *Phys. Rev. Lett.*, 76(10):1631–1634, 1996.

[37] M. I. Tribelsky and M. G. Velarde. Short-wavelength instability in systems with slow long-wavelength dynamics. *Phys. Rev. E*, 54(5):4973–4981, 1996.

[38] R. W. Wittenberg. Large-scale contributions to spatiotemporal chaos. In preparation.

[39] R. W. Wittenberg. *Local Dynamics and Spatiotemporal Chaos. The Kuramoto-Sivashinsky Equation: A Case Study.* PhD thesis, Princeton University, 1998.

[40] R. W. Wittenberg. Kuramoto-Sivashinsky equation. In *Encyclopaedia of Mathematics*, Supplement III. Kluwer Academic Publishers, 2001 (to appear).

[41] R. W. Wittenberg. Dissipativity, analyticity and viscous shocks in the (de)stabilized Kuramoto-Sivashinsky equation. To appear in *Physics Letters A*, 2002.

[42] R. W. Wittenberg and P. Holmes. The limited effectiveness of normal forms: A critical review and extension of local bifurcation studies of the Brusselator PDE. *Physica D*, 100:1–40, 1997.

[43] R. W. Wittenberg and P. Holmes. Scale and space localization in the Kuramoto-Sivashinsky equation. *Chaos*, 9(2):452–465, 1999.

[44] R. W. Wittenberg and P. Holmes. Spatially localized models of extended systems. *Nonlinear Dynamics*, 25:111–132, 2001.

[45] H. Xi, R. Toral, J. D. Gunton, and M. I. Tribelsky. Extensive chaos in the Nikolaevskii model. *Phys. Rev. E*, 62(1):R17–R20, 2000.

[46] G. Xiong, J. D. Gunton, H. Xi, and R. Toral. Space and scale localization in a model of a direct transition to spatiotemporal chaos. Preprint nlin.CD/0105036, 2001.

[47] V. Yakhot. Large-scale properties of unstable systems governed by the Kuramoto-Sivashinski equation. *Phys. Rev. A*, 24(1):642–644, 1981.

[48] S. Zaleski. A stochastic model for the large scale dynamics of some fluctuating interfaces. *Physica D*, 34:427–438, 1989.